REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

29 [C, D].—LEON KENNEDY, *Handbook of Trigonometric Functions—Introducing Doversines*, Iowa State University Press, Ames, Iowa 1961, 397 p., 23.5 cm. Price $6.50.

In his preface the author states that this new set of trigonometric tables has been arranged specifically to facilitate the solution of both plane and spherical triangles. The term "doversine" has been introduced as an abbreviation for "doubled versed sine."

The introduction to these tables contains a derivation of the law of haversines for spherical triangles and of the "law of doversines" for plane triangles. Also included therein are a table of algebraic signs for all the tabulated trigonometric functions for the four quadrants; a composite graph of the cosine, versine, haversine, and doversine; a compilation of fundamental trigonometric formulas and identities; and a presentation of formulas for solving plane and spherical triangles, including the navigational triangle.

The main table, occupying 360 pages, consists of six-figure natural and logarithmic values of the six standard trigonometric functions and of the versine and doversine. These data are conveniently displayed on facing pages for each sexagesimal degree to 180 degrees at increments of a minute in the argument. Natural values and their logarithms appear in adjacent columns, printed in black and blue, respectively. Differences are not tabulated.

Following this table, there is a table of six-place mantissas of the common logarithms of the integers between 1000 and 10,000.

The numerical values presented in these original tables were computed by the author on an IBM 650 system in the Ames Laboratory at Iowa State University. The doversines and their logarithms were previously computed on the ILLIAC, and these results were compared with corresponding data obtained in the later calculation.

The reviewer has compared nearly forty percent of the tabular entries appearing in the first quarter of the main table with the corresponding entries appearing in more elaborate tables such as those of J. Peters. The reliability of these trigonometric values and their logarithms may be inferred from the fact that such careful examination revealed just one typographical error (in csc $8°48'$) and one terminal-digit error (in log sec $5°5'$). On the other hand, the following five rounding errors were discovered by the reviewer in the table of logarithms of numbers: log 499, *for* 698100, *read* 698101; log 2443, *for* 387924, *read* 387923; log 8652, *for* 937116, *read* 937117; log 8854, *for* 947139, *read* 947140; and log 8884, *for* 948608, *read* 948609.

Correspondence with the author has disclosed that these last errors are attributable to a programming error, which led to the omission of one of the tests for retention of an adequate number of terms in the evaluation of the logarithms by power series. The publishers have recently informed the reviewer that they will make appropriate corrections in the future printings.

This handbook of trigonometric functions is unique among the tables that this reviewer has examined, particularly because of its format, which is to be contrasted with the customary semiquadrantal arrangement. The tabulation of the versine and doversine clearly requires the extended range of the argument given in these tables. The author justifies the tabulation of the six standard trigonometric functions also over the first two quadrants on the basis of the resulting ease of application to the solution of triangles. The typography is generally excellent, and the arrangement of the tabular data convenient. This book should prove a useful addition to the literature of mathematical tables.

J. W. W.

30 [F].—R. Kortum & G. McNiel, *A Table of Periodic Continued Fractions*, Lockheed Aircraft Corporation, Sunnyvale, California, 1961, xv + 1484 p., 29 cm.

This huge and interesting table contains, first, the *half-period* of the regular continued fraction for the $\sqrt{D}$ for each non-square natural number D less than 10,000. For example, since

$$\sqrt{13} = 3 + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{3 + \sqrt{13}},$$

under $D = 13$ are listed the *partial quotients*: 3, 1, 1. Again, since

$$\sqrt{19} = 4 + \frac{1}{2} + \frac{1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{2} + \frac{1}{4 + \sqrt{19}},$$

under $D = 19$ is the list: 4, 2, 1, 3.

Next, let

$$\sqrt{D} = q_0 + \frac{1}{q_1 +} \frac{1}{q_2 +} \cdots \quad \text{and} \quad x_i = q_i + \frac{1}{q_{i+1} +} \cdots$$

and $x_i = (\sqrt{D} + P_i)/Q_i$. Then Q_i, the *denominators of the complete quotients*, are listed in a row parallel to the q_i.

If p, the *period* of the continued fraction, is odd, as in $p = 5$ for $D = 13$, the table gives the smallest solution x, y of

$$x^2 - Dy^2 = -1.$$

If p is even, as in $p = 6$ for $D = 19$, the smallest solution is given of the so-called Pell equation:

$$x^2 - Dy^2 = +1.$$

The values of D for which p is odd are marked with an asterisk.

Finally, if $p^2/D > 1$, this ratio is given to 9 decimals. All of this was computed on an IBM 7090 in 36 minutes.

While in [1] the continued fractions for $D < 10{,}011$ have already been given, together with *both* sequences P_i and Q_i defined above, the range here of D for the solutions x, y, and for the ratio p^2/D, would appear to exceed that in any published table.

From a theoretical point of view the quantity p^2/D is of considerable interest. If we list those D where p^2/D attains a new maximum we obtain the following table: